Please staple this cover sheet in front of your answers. (Behind Department of Physics coversheet.)

NAME: **ID#:**

ELEC441: Assignment 7

Due 4pm Wednesday 5th May 2021

https://amoqt.otago.ac.nz/people/asbradley/elec441

1. Show that $\left|\phi^e_{ff}(\tau)\right| \le \phi^e_{ff}(0)$, $\forall \tau$.

2. Show that $\Phi^p_{ff}(\nu)$ is the power spectral density of a signal with finite average power, $f(t)$. Do this by passing the signal through an ideal bandpass filter, and then making an average power measurement.

3. A random telegraph signal with values ± 1, shown in the figure, can be found in the file `telegraph.txt`, by opening the link `https://www.dropbox.com/s/37yjij419crcx5l/a7_data.zip?dl=0`.

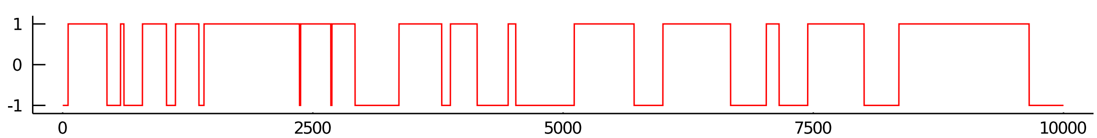

Figure 1: Random telegraph signal

The data is buried in a noisy signal (`signal.txt` from the same link) shown in the figure below. The data has been encoded in the noise by first shifting (delaying) the telegraph signal, and then adding noise. Use a matched filter to find the value of the shift (an integer between 1 and 20000). Attach a file showing your code, and the result of applying the filter.

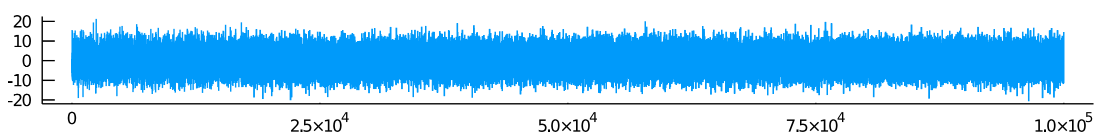

Figure 2: Signal resulting from transmitting the telegraph signal in a noisy environment.

You may find the following julia code useful:

```
using FFTW

"""
      y = matchedfilter(s,x,j=0)

Apply the Middleton-North matched filter to data `x`, with target signal `s`.
`j=0` matches peak power at the first element of `s`, or a delay of zero.
"""
function matchedfilter(s,x,j=0)
  h = zero.(x)
  h[1:length(s)] .= s
  h = h |> reverse |> conj
  h = circshift(h,j)
  H = fft(h)
  X = fft(x)
  return ifft(H.*X)
end
```

4. Consider a digital signal in the form of a *random bitstream* (logical bit values `true/false` represented by ± 1, each lasting a fixed duration T). We can view the fundamental data is a sampled digital signal $d(t) = \sum_{k=-\infty}^{\infty} c_k \delta(t - kT)$, where $c_k \in \{-1, +1\}$ with equal probability. The transmitted signal may be expressed as $x(t) = (d * h)(t)$, where $h(t)$ is the impulse response of the a linear system performing the signal encoding. Show that the encoded signal has significant power at zero frequency.

   [***Hint:*** Take the Fourier transform of $d(t)$. The average power spectral density is easily found for a stream of N bits, each lasting time T, since $\mathrm{E}(c_k c_j) = \delta_{jk}$.]

   One way to get round this is to use *Manchester encoding* where a logical `true` is represented by a signal at +1 for the first half of the bit interval and -1 for the second half. A logical `false` is represented by -1 for the first half of the bit interval and +1 for the second half, as shown in the figure.

   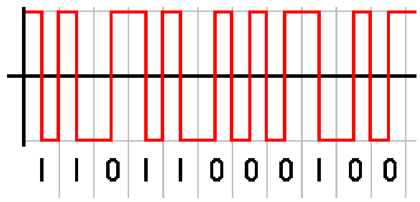

   Image from Wikipedia

   Calculate the power spectrum of a random bitstream with clock frequency $(1/T)$ = 1 MHz that has been Manchester encoded. Plot it along with a power spectrum calculated from simulated data. Include a printout of your code.

SCORE: