

Please staple this cover sheet in front of your answers. (Behind Department of Physics coversheet.)

NAME:

ID#:

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## ELEC441: Assignment 5

### Due 1pm Friday 16th April 2021

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<https://amoqt.otago.ac.nz/people/asbradley/elec441>

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1. Give the definition of the Fourier transform of a generalized function in terms of its action on test functions. Show formally that the Fourier transform of  $f(t) = t$  is

$$F(\nu) = \frac{j\delta'(\nu)}{2\pi}.$$

(Points will be given for how easy your proof is to follow).

2. For integer  $k > 0$ , what is the Fourier transform of  $t^k$ ? Show this without using test functions, starting from the FT pair  $1 \longleftrightarrow \delta(\nu)$ .
3. If the Fourier transform of a generalized function  $g(t)$  is  $G(\nu)$ . Use test functions to show that

$$g(at) \longleftrightarrow \frac{1}{|a|} G\left(\frac{\nu}{a}\right).$$

4. A band-limited signal  $f(t)$ , is sampled every  $T$  to create  $f_s(t)$ , where  $T$  is chosen so that the Nyquist criterion is satisfied. Suppose that rather than using the exact sinc-based interpolation technique to recover the original signal, a reconstruction is made using linear interpolation between samples.

- (a) Show how linear interpolation between samples can be achieved by convolving  $f(t)$  with an appropriate function. Graph the impulse response of a linear interpolator.

*Hint: Linear interpolation is LTI. Consider the response to an impulse: how does a linear interpolator respond to a sampled input signal with only one non-zero point?*

- (b) Describe the difference between  $F(\nu)$  and the Fourier transform of the linearly interpolated approximation.

5. Imagine that you are trying to explain aliasing to someone.

- (a) Show that  $f(t) = \sin(2\pi\nu_0 t)$  and  $g(t) = \sin(2\pi(\nu_0 + \frac{1}{T})t)$ , are equal when  $t$  is an integer multiple of  $T$ .
- (b) Using either julia, python, MATLAB or something else, make a plot demonstrating this using  $\nu_0 = 0.1$ ,  $T = 1$ . Attach the code and plot to your assignment.

(P.S. I love open source, you will get 1/4 of an extra mark for using something open source)

**SCORE:**